Chapter 3: Probability
3.1: Basic Concepts of Probability

Objectives
• Identify the sample space of a probability experiment and a simple event
• Use the Fundamental Counting Principle
• Distinguish classical probability, empirical probability, and subjective probability
• Determine the probability of the complement of an event
• Use a tree diagram and the Fundamental Counting Principle to find probabilities

Probability Experiments

Probability experiment
• An action, or trial, through which specific results are obtained.
Example: counts, measurements, or responses are obtained

Outcome
• The result of a _____________ trial in a probability experiment.

Sample Space
• The set of ______ outcomes of a probability experiment.

Event
• Consists of one or more _______________ and is a subset of the sample space.

Example of a Probability Experiment:

<table>
<thead>
<tr>
<th>Probability Experiment</th>
<th>Roll a die</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outcome</td>
<td>3</td>
</tr>
<tr>
<td>Sample Space</td>
<td>{1, 2, 3, 4, 5, 6}</td>
</tr>
<tr>
<td>Event</td>
<td>&lt;Die is even&gt; = {2, 4, 6}</td>
</tr>
</tbody>
</table>
Example: Identifying the Sample Space

A probability experiment consists of tossing a coin and then rolling a six-sided die. Describe the sample space.

Solution:

One way to list outcomes for actions occurring in a sequence is to use a tree diagram.

Tree diagram:

Description:

Simple Events

<table>
<thead>
<tr>
<th>Simple event</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple event</td>
<td>An event that consists of a single outcome. Example:</td>
</tr>
<tr>
<td></td>
<td>An event that consists of more than one outcome is not a simple event.</td>
</tr>
<tr>
<td></td>
<td>Example</td>
</tr>
</tbody>
</table>

Example: Identifying Simple Events

Determine whether the event is simple or not.

- You roll a six-sided die. Event B is rolling at least a 4.

Solution:
Fundamental Counting Principle

If ___ can occur in \( m \) ways and a ____ can occur in \( n \) ways, the number of ways the two events can occur in _________________ is:

- Can be extended for any number of events occurring in sequence.

Example: Fundamental Counting Principle

You are purchasing a new car. The possibilities/ways:
- Event: (Manufacturer):Ways: (Ford, GM, Honda)
- Event: (Car size): Ways: (compact, midsize)
- Event: (Color): Ways: (white (W), red (R), black (B), green (G))

How many different ways can you select one manufacturer, one car size, and one color? Use a tree diagram to check your result.

Using the Fundamental Counting Principle:

Using a Tree Diagram:

Types of Probability

Classical (theoretical) Probability
- Each outcome in a sample space is equally likely.

Example: Finding Classical Probabilities
You roll a six-sided die. Find the probability of each event.

1. Event A: rolling a 3
2. Event B: rolling a 7
3. Event C: rolling a number less than 5
Empirical (statistical) Probability

- Based on ______________ obtained from probability

- ______________

- ______________ ________________ of an event – ______________

Example: Finding Empirical Probabilities

A company is conducting an online survey of randomly selected individuals to determine if traffic congestion is a problem in their community. So far, 320 people have responded to the survey. What is the probability that the next person that responds to the survey says that traffic congestion is a serious problem in their community?

<table>
<thead>
<tr>
<th>Response</th>
<th>Number of times, f</th>
</tr>
</thead>
<tbody>
<tr>
<td>Serious problem</td>
<td>123</td>
</tr>
<tr>
<td>Moderate problem</td>
<td>115</td>
</tr>
<tr>
<td>Not a problem</td>
<td>82</td>
</tr>
</tbody>
</table>

Law of Large Numbers
Types of Probability

Subjective Probability

- Intuition, educated guesses, and estimates.

Example:

Example: Classifying Types of Probability

Classify the statement as an example of classical, empirical, or subjective probability.

1. The probability that you will be married by age 30 is 0.50.

2. The probability that a voter chosen at random will vote Republican is 0.45.

3. The probability of winning a 1000-ticket raffle with one ticket is \( \frac{1}{1000} \).

Range of Probabilities Rule

Range of probabilities rule

- The probability of an event \( E \) is between _______ and _____, inclusive.

<table>
<thead>
<tr>
<th>Impossible</th>
<th>Unlikely</th>
<th>Even chance</th>
<th>Likely</th>
<th>Certain</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.5</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Complementary Events

Complement of event $E$

- The set of all outcomes in a sample space that are not included in event $E$.

Example: Probability of the Complement of Event

You survey a sample of 1000 employees at a company and record the age of each. Find the probability of randomly choosing an employee who is not between 25 and 34 years old.

- Use empirical probability to find $P(\text{age 25 to 34})$

- Use the complement rule

<table>
<thead>
<tr>
<th>Employee ages</th>
<th>Frequency, $f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>15 to 24</td>
<td>54</td>
</tr>
<tr>
<td>25 to 34</td>
<td>366</td>
</tr>
<tr>
<td>35 to 44</td>
<td>233</td>
</tr>
<tr>
<td>45 to 54</td>
<td>180</td>
</tr>
<tr>
<td>55 to 64</td>
<td>125</td>
</tr>
<tr>
<td>65 and over</td>
<td>42</td>
</tr>
</tbody>
</table>

Example: Probability Using a Tree Diagram

A probability experiment consists of tossing a coin and spinning a fair spinner shown. Use a tree diagram to find the probability of tossing a tail and spinning an odd number.

Tree Diagram:

$$P(\text{tossing a tail and spinning an odd number}) =$$
Example: Probability Using the Fundamental Counting Principle

Your student identification number consists of 6 digits. Each digit can be 0 through 9 and each digit can be repeated. What is the probability of getting your student identification number when randomly generating six digits?

Solution: Probability Using the Fundamental Counting Principle

\[ P(\text{your ID number}) = \]